The Mechanics of Hydraulic Fractures in Shales[1]

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ABSTRACT

The Oak Ridge National Laboratory is now using an adaptation of the oil-field technique of hydraulic fracturing for the disposal of some of its liquid radioactive wastes. In this adaptation, the aqueous wastes are mixed with Portland cement and other solids and then pumped into a hydraulically induced horizontal fracture in shale rocks at about 1000-ft depth. Since 1966 about 515,000 gal of waste containing more than 215,000 curies of activity have been disposed of by this method.

The rocks overlying the solidified grout sheets provide the primary isolation and containment barrier for the wastes, and their integrity must be preserved. The development of the hydraulic fracturing waste disposal method therefore required an examination of both the mechanics of the induced fractures and the influence which each injection has on the surrounding rocks. These investigations indicated that the orientation of fractures are apparently controlled by both the state of stress in the ground and the anisotropy, that is, the bedding planes, of the shales. However, each injection imposes a stress field on the rocks which results in an uplift of the surface over the grout sheets and at the same time contributes toward the creation of a stress condition where some future fracture may be vertical instead of horizontal.

Although hydraulic fracturing has been an extremely successful waste-disposal method at Oak Ridge, it does not follow automatically that it can be used at any other location. The technique is predicted upon the formation of horizontal rather than vertical fractures, and each proposed site must be extensively tested in order to prove that the fractures are indeed horizontal, or at least conformable in a nearly horizontal formation. Unequivocal proof of fracture orientation will require several core holes from the surface and intersecting the grout sheets, but a number of techniques can be used to obtain presumptive evidence of fracture orientation, thereby reducing the number of core holes required. Some of these, including

1. *analysis of fluid pressures at breakdown, and during and after the injection;*
2. *measurement of surface uplift; and*
3. *the detection of micro-seismic signals generated by the propagating tip of the fracture,*

will be tested during an experimental site examination in western New York state during the coming summer.

INTRODUCTION

The Oak Ridge National Laboratory began work about 1960 on a system for the disposal of radioactive waste solutions by a technique based on hydraulic fracturing (de Laguna, 1968). The research and development program was successfully completed in 1966; and, since that time, about 515,000 gal of waste containing more than 215,000 curies of activity have been disposed of on a routine basis at Oak Ridge. In this method, the aqueous wastes are first mixed with preblended dry solids containing Portland cement, various clays, fly-ash, and certain additives (Tamura, 1967). The

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resulting slurry is then pumped down a well and out into a conformable, nearly horizontal fracture in the thick shale formation at a depth of about 1000 ft (Fig. 1). The well is prepared for the injection by cutting a slot in the casing at the desired depth and pressurizing the well with water. This induces a fracture in the rocks into which the slurry is pumped, causing the fracture to extend. After completion of the pumping phase, the well is shut in and the slurry is allowed to harden, under pressure, to form thin horizontal grout sheet. This procedure is repeated at successive intervals up the well, creating a stack of horizontal grout sheets.

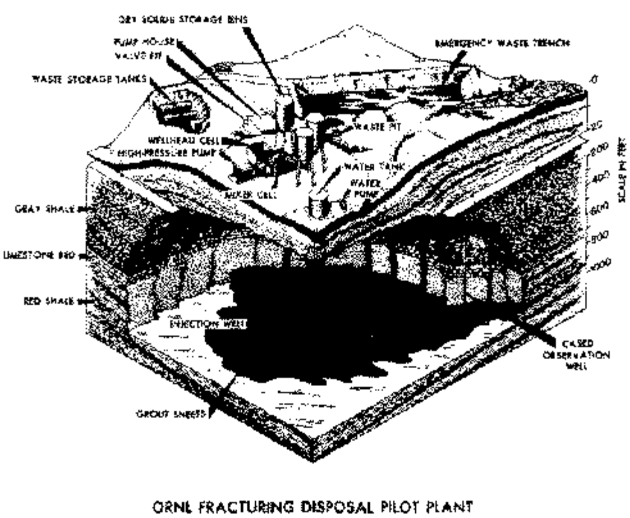

Figure 1. Pictorial representation of the hydraulic fracturing plant and the injected grout sheets.

The unique advantage of this system is that it completely and permanently removes the radionuclides from any possible accidental release to the biological environment. This is accomplished by the fact that both shielding and containment are supplied by approximately 1000 ft of rock cover overlying the grout sheets. The radionuclides are deposited and immobilized in the impermeable shale, far below any circulating groundwater, and are retained in a nearly unleachable grout sheet.

The success of this technique depends upon the formation of horizontal rather than vertical fractures when the waste slurries are pumped underground. A vertical fracture would represent a breach of the containment and isolation barrier provided by the overlying rocks. Therefore, a significant portion of the present research program is devoted to the development of an understanding of the behavior of the underground fractures, especially as related to their orientation. This paper is a summary of the mechanics of hydraulic fracturing in shale emerging out of this research program.

PRELIMINARY EXPERIMENTAL RESULTS

The horizontality of the hydraulically induced fractures at Oak Ridge was confirmed by the core drilling which was conducted as part of the experimental program (de Laguna, 1968). In an early test, two grout sheets, each identified by a chemical dye and a radioisotope tag, were injected at depths of about 930 ft and 700 ft. The location and shape of the injected grout sheets were subsequently determined by drilling 24 core holes in the area (Figs. 2 and 3). Correlation of the depths to

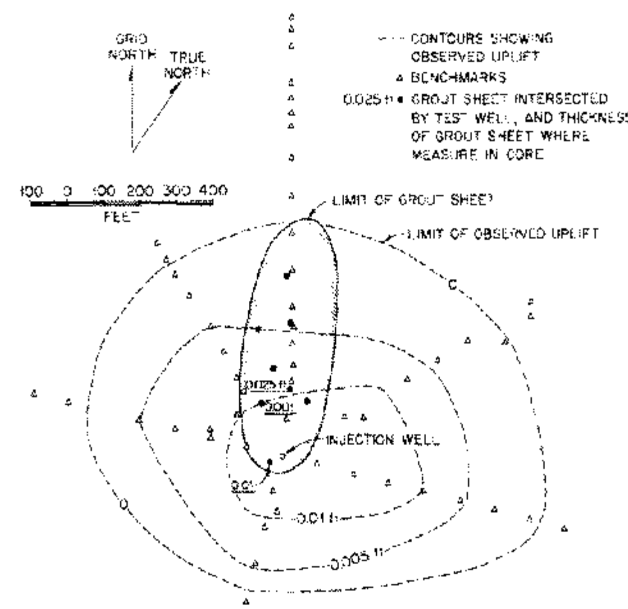

Figure 2. Extent and thickness of lower grout sheet with its associated surface uplift.

the grout sheets confirmed that the fractures are, in general, conformable in the nearly horizontal shales (Fig. 4). Detailed examination of these and other cores containing the grout sheets indicated that the fractures tended to cut across the bedding whenever a small drag fold was encountered. This resulted in a slight (stratigraphic) upward drift of

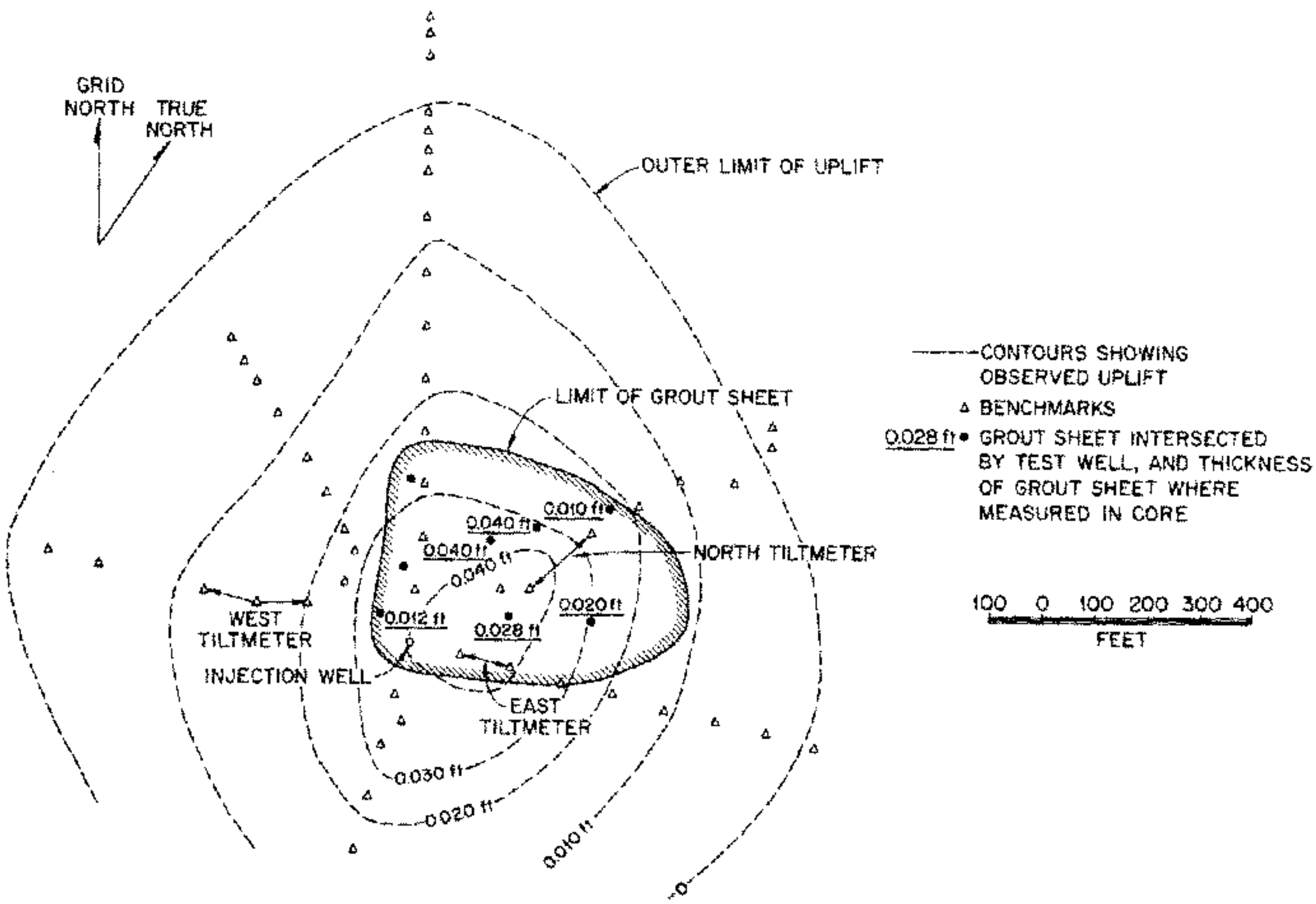

Figure 3. Extent and thickness of upper grout sheet with its associated surface uplift.

the fractures amounting to probably less than 1 ft per 100 ft of distance from the injection well.

A part of this experiment involved measuring the uplift of the ground surface resulting from the grout injections. This was done by repeated levelings on a system of bench marks in the area (Figs. 2 and 3). These measurements indicated than an area much larger than the extent of the grout sheets had been domed upward slightly with the maximum uplift centered at the injection well and amounted to a few tenths of an inch or approximately the thickness of the grout sheets. These uplift measurements and the information on the actual size and shape of the grout sheets represented the principal basic data concerning the mechanics of the fracturing. A detailed analysis of these data was carried out, (McClain, 1968) and it was concluded that, as a first approximation, the shale can be assumed to deform elastically and that the injected grout sheet can be idealized as a thin circular ellipsoid.

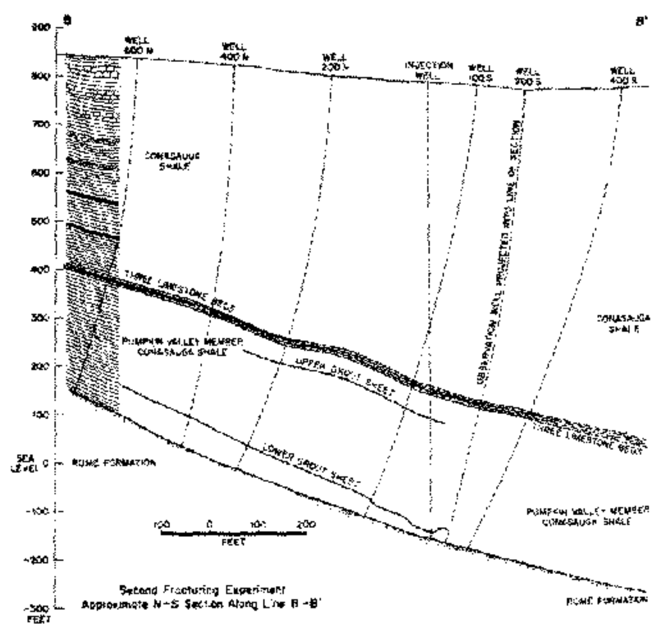

Figure 4. Vertical section showing two test grout sheets.

FACTORS EFFECTING FRACTURE ORIENTATION

Hydraulic fracturing in shales differs from the usual oil-field operations in that the fractured material has essentially no permeability. This greatly simplifies the theoretical situation, since it is not necessary to consider the effects of pore pressures and their gradients. Also the volume of the fracture can be assumed to be equal to the volume of fluid injected. For a thin circular fracture, the minimum internal pressure required to cause extension of the fracture is given by Sack (1946) to be:

$$P_o = \left[\frac{\pi \lambda E}{s(1-\nu^2)c}\right]^{1/2} \tag{1}$$

where

P_o = minimum fracture extension pressure in excess of the confining pressure (psi) perpendicular to the plane of the fracture,

λ = specific surface energy of the rock (ft-lb/in.^{2}),

E = modulus of elasticity (psi),

ν = Poisson's ratio,

c = radius of the circular fracture (ft).

For a fracture radius (c) greater than 100 ft, this fracture extension pressure (P_o) is very small, less than 10 psi. The specific surface energy (λ) of a rock is related to its tensional strength, and shales are, of course, notorious for their strength anisotropy. This is especially true of the Conasauga shales at Oak Ridge which have essentially no strength parallel to the bedding. However, this strength anisotropy would have only a very small influence on the fracture orientation, since only a very small internal pressure is required to overcome it as seen by Eq. (1). This line of reasoning leads to the conclusion that the orientation of the fractures is controlled primarily by the state of stress in the ground, with, in this case, a very minor second order influence exercised by the bedding planes in the shale. This, of course, was the conclusion reached earlier by Hubbert and Willis (1957); hydraulic fractures will be formed perpendicular to the direction of the least compressive principal stress.

For the Oak Ridge area, this implies that the vertical overburden stress is less than the horizontal compressive stresses. Furthermore, this view of the situation is completely consistent with the field observations. Throughout most of the fracture, the least compressive stress must be either exactly perpendicular to the bedding or close enough that the strength anisotropy would make up the difference, thereby producing fractures which are conformable in the shale. At the small drag folds, the more steeply dipping bedding planes would no longer be perpendicular to the least stress, and the fractures, being controlled by the stress, would cut across the bedding.

Based on this interpretation of the general features of the process, it is possible to examine in greater detail the mechanics of the various phases of a hydraulic fracturing operation, specifically, fracture initiation, extension of the fracture, and the net effect of the completed grout sheet on the surrounding rocks.

ANALYSIS OF FRACTURE MECHANICS

During the initiation of the fracture (formation "breakdown" in oil field practice) and until the fracture has been extended beyond the area immediately adjacent to the injection well, the situation is much more complicated than that described above. The very presence of the well results in stress concentrations, and the influence of the strength anisotropy of the shales is much larger. Both of these effects are such as to favor the initiation of a horizontal rather than a vertical fracture. However, their influence dies out very rapidly with increased distance from well, and it has been suggested (Kehle, 1964) that under certain conditions horizontal fracture could be initiated which would then revert to its "normal" stress-controlled vertical mode and back up into the well. For waste-disposal operations, the situation at fracture initiation is even further complicated by the sand-erosion technique which is used to cut a slot in the well casing and out into the shale formation. The exact configuration of this slot can only be guessed at, but it is almost certainly quite irregular and varies considerably from one slot to the next. Probably as a result of this variability, an extremely wide range of breakdown pressures have been measured on the numerous fracturing operations at Oak Ridge. However, all breakdown pressures have been larger than the calculated overburden stress.

Fortunately, the fracture initiation process has not been a major concern in this development program, because the fractures are extended to distances of several hundred feet in the course of injecting about 100,000 gal of waste. Therefore, no detailed treatment of fracture initiation has been undertaken.

Fracture extension.

The pressure relationships in a thin, circular, horizontal, axially symmetrical fracture are shown in Figure 5 where P_w is the fluid pressure measured at the center of the injection well during the pumping operations, h_e is the pressure loss in the turbulent flow region resulting from the entrance and friction losses, and h_f is the friction loss in the remainder of the fracture where the flow is laminar. The fluid pressures at the various locations indicated on Figure 5 can be expressed:

$$P_3 = \sigma + P_o$$

$$P_2 + \frac{\gamma V_2^2}{2g} = P_3 + h_f$$

$$P_w = P_1 = P_2 + \frac{\gamma V_2^2}{2g} + h_e \qquad (2)$$

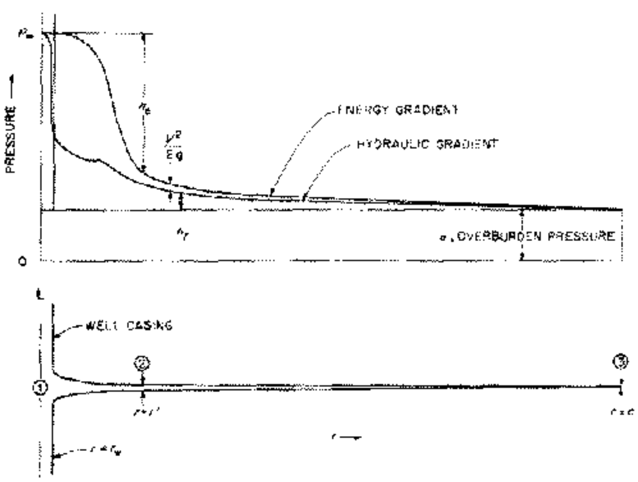

Figure 5. Pressure relationships (above) in thin circular fracture (below).

where

σ = vertical or overburden stress (psi),

P_o = minimum fracture extension pressure (psi),

V = bulk velocity of fluid (ft/sec),

g = acceleration of gravity (32.2 ft/sec^2),

γ = density factor of the fluid (psi/ft).

Since the fracture is assumed to be horizontal, there is no difference in elevation between the center of the well and the tip of the fracture and the overburden pressure (σ) can be taken as constant. The minimum fracture extension pressure (P_o) is so small, compared to the overburden pressure, that it can be neglected. The radial velocity at the center of the injection well (r = O) is, from symmetry, equal to zero. Since this analysis will be concerned with those cases where the fracture has extended some distance from the injection well, that is, where the velocity of the fracture tip is greatly reduced, its velocity head will be approximately equal to zero. Equation (2) is therefore reduced to

$$P_w = \sigma + h_e + h_f. \qquad (3)$$

Since the overburden pressure (σ) in Eq. (3) can be estimated satisfactorily from the depth, it remains only to evaluate the two components of the losses h_e and h_f. The zone of turbulent flow can be defined as the region where Reynold's number (N_R) is greater than 2×10^3. Since the hydraulic radius (R_H) for flow between parallel plates is

$$R_H = \frac{\alpha}{2} \qquad (4)$$

where

α = separation of the plates or, in this case, the average thickness of the fracture, the Reynold's number will be given by

$$N_R = \frac{4 R_H V \rho}{\mu} = \frac{2\alpha V \rho}{\mu} \qquad (5)$$

where

ρ = bulk density in lb-sec^2/ft^4 and

μ = absolute viscosity of fluid in lb-sec/ft^2.

From the geometry of axially symmetrical radial flow, the bulk velocity as a function of radial distance (r) will be

$$V = \frac{Q}{2\pi\alpha r} \qquad (6)$$

where

Q = flow rate (ft^3/sec). Combining Eqs. (5) and (6), the expression

$$N_R = \frac{Q \rho}{\pi r \mu} \qquad (7)$$

is obtained, which indicates that for this situation Reynold's number is not a function of crack thickness (α). The limit of the turbulent zone will therefore be given by

$$r' = \frac{Q\rho}{2000\,\pi\mu} \tag{8}$$

The friction losses (h_f) in the laminar region ($r' \leqslant r \leqslant c$) can be found by examining the pressure drop (dP) across a "ring" of differential width *dr*, with the assumption that the fluid is moving between parallel plates of separation α. This pressure differential can be written

$$dP = \frac{\gamma}{2g}(V^2{}_{r+dr}) - \frac{\gamma}{2g}V_r^{\,2} + \frac{fV^2\rho}{288\,R_H} \tag{9}$$

where

f = fanning friction factor.

In order to be strictly correct, Eq. (9) should include a component arising from the viscosity effects of axial flow. This factor results from the energy required to increase the horizontal distance separating two points as they move from one side of the differential element to the other. This is a second order viscosity effect, and it has been neglected.

Since the friction factor for laminar flow between parallel plates is

$$f = \frac{24}{N_R} = \frac{12\mu}{\alpha V \rho} \tag{10}$$

and, in the laminar flow region, the velocity heads are insignificant, Eq. (9) reduces to

$$dP = \frac{\mu Q}{24\,\pi\alpha^3\,r}\;dr. \tag{11}$$

The pressure at $r = r'$ will then be the integral of Eq. (11) between r' and the tip of the fracture (c).

$$P_{r'} - o = h_f = \int_{r'}^{c} \frac{\mu Q}{24\,\pi\alpha^3\,r}\,dr \tag{12}$$

$$h_f = \frac{\mu Q}{24\,\pi\alpha^3}\;\ln\frac{c}{r'}$$

Sneddon (1946) has shown that a flat, circular crack with internal pressure in an infinite elastic body will assume an elliptical cross section and that the center thickness of this flattened ellipsoid (W_{max}) will be a function of the average pressure (P_a) regardless of the actual pressure distribution in the fracture, according to

$$W_{max} = \frac{8(1-\nu^2)c\,P_a}{\pi E} \tag{13}$$

The influence of the ground surface is ignored by the assumption of an infinite (rather than semi-infinite) elastic body. This simplification is justified by the fact that it has been demonstrated that, for horizontal fractures with radii less than four-thirds of the depth, the solution for infinite and semi-infinite medias are essentially identical (Kern and Perkins, 1961). Since the turbulent region extends only a few feet from the well, its additional contribution to the average pressure in the fracture (P_a) can be neglected. The average pressure can now be computed from Eq. (12) by observing that $h_f = O$ at the tip of the fracture ($r = c$), according to

$$P_a = \frac{\pi r'^2 h_f + 2\pi h_f \displaystyle\int_{r'}^{c} r\,dr - \frac{2Q}{24\,\pi\alpha^3}\int_{r'}^{c} r\ln\frac{r}{r'}\,dr}{\pi c^2} \tag{14}$$

which reduces to

$$P_a = \frac{\mu Q}{48\,\pi\alpha^3}\quad(1 - r'^2/c^2)\,. \tag{15}$$

Since c will always be much larger than r', the value of the quantity in brackets will be approximately unity and can be dropped. The geometry of an ellipsoid gives the relationship between the maximum width (W_{max}) and the average width (α) in a direction perpendicular to an axial plane:

$$W_{max} = 3\;\alpha/2. \tag{16}$$

Equations (12), (14), (15), and (16) can now be combined to yield the expressions

$$h_f = \frac{\sqrt{3}}{8}\;\ln\frac{c}{r'}[E/(1-\nu^2)\,c]^{3/4}\,[\pi^2\mu Q]^{1/4} \tag{17}$$

$$W_{max} = \frac{\sqrt{3}}{2}[(1-\nu^2)\;c\mu Q/\pi^2 E]^{\;1/4}, \text{ and} \tag{18}$$

$$c = [9\,E\,\Delta^4/\pi^2\;(1-\nu^2)\,\mu Q]^{1/9} \tag{19}$$

where

Δ = cumulative volume injected (ft^3).

It should be recognized that the center thickness of the fracture, defined by Eq. (18), is the theoretical separation of the two sides of the fracture at the well due to the internal fluid pressures. The actual thickness of the slot at the well and for a short distance inside the fracture will be much larger as a result of the mechanical erosion of both the sand-jet slotting operation and the high velocity fluid flows at that point. Because of this, the evaluation of the losses in the turbulent region (h_e) is much more difficult than for the laminar zone. In relatively simple hydraulic systems, such as the fluid in a tank exiting through a small horizontal pipe, the entrance losses can be defined by

$$h_e = K_e \, (\gamma \, V^2/2\,g) \tag{20}$$

where the coefficient K_e has a value between 0 and 1.0, depending on the geometrical configuration of the entrance pipe. The value of K_e is known only for the very simplest systems, and usually it is necessary to evaluate K_e by experiment. In hydraulic fracturing, it is impossible to make a reasonable estimate of the shape of the fracture just outside the well casing. Even if the value of the entrance loss coefficient (K_e) could be estimated, there would remain the problem of selecting the proper velocity (V) to use in Eq. (20). One way around these difficulties is to assume that the proper value of the velocity for use in Eq. (20) occurs at some characteristic distance from the well for each flow rate. It is then possible to write the generalized relationship:

$$V = A Q^m \Delta^n \tag{21}$$

where A is an arbitrary constant and the Δ^n component grows out of the fact that part of the thickness of the fracture [α in Eq. (6)] is a function of the total volume injected (Δ). Substitution of Eq. (21) into Eq. (20) produces the expression

$$h_e = \frac{\gamma}{2\,g} \, B \, Q^{2m} \, \Delta^{2n} \tag{22}$$

where

$$B = A^2 K_e.$$

Experimental test of pressure analysis.

Substitution of Eq. (17) for the laminar zone losses and Eq. (22) for the turbulent zone losses into Eq. (3) produces an expression describing the variation of the pressure in the well during the pumping operations. According to this expression the pressure measured in the well should decrease during the course of the injection as a power function of the cumulative volume injected. In order to test this interpretation of the fracture dynamics, and to obtain information on the constants describing the turbulent zone losses [Eq. (22)], a special water injection test was conducted. It was not possible to use data from the routine grout injections for this purpose, because the cement-based slurry behaves as a non-Newtonian fluid and the analysis would not apply. In this water-injection test, accurately metered flow rates of 40, 100, and 250 gpm were maintained in sequence for periods of 10 to 40 min each. The results of this test are shown on Figure 6 where the injection

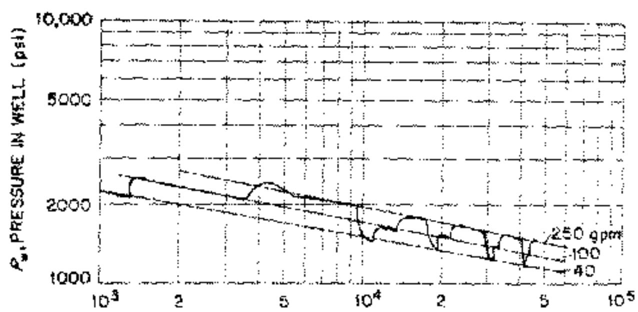

Figure 6. Pressure as a function of total volume injected for water injection test.

pressure (reduced to the elevation of the fracture) is shown as a function of the cumulative volume injected. The most significant aspect of these results is the power function decrease in injection pressure, for each of the flow rates, as the volume increases. This is exactly the behavior predicted by the theoretical analysis. Another interesting feature of the results shown on Figure 6 is the flow transients associated with each change of flow rate. These transients are also predicted in that the thickness of the fracture [W_{max} in Eq. (18)] is greater for higher flow rates (Q). Therefore, when the pumping rate is suddenly increased from say 100 to 250 gpm, equilibrium conditions will not be reached until the increased volume in the fracture has been "filled-up."

The data from this test were analyzed, by least squares curve fitting, to obtain the turbulent zone constants for Eq. (22), giving

$$h_e = \frac{\gamma}{2g} (1.12 \times 10^6) \; Q^{2\,(0.07)} \Delta^{2\,(-0.09)} \qquad (23)$$

Unfortunately, this one set of constants will not permit a quantitative interpretation of any future fracturing operations because of the expected variability between any two slots in the casing and therefore the variability in the turbulent losses of two injections. This difficulty could be overcome by devising some method of measuring the fluid pressure in the fracture at some point beyond the turbulent zone. However, it is felt that the analysis aids considerably in understanding the mechanics of the fracture extension process.

Pressure relationships in vertical fractures.

This type of analysis can be made for fractures with shapes other than circular (McClain, 1968b). Perhaps more importantly, the analysis can be extended to include nonhorizontal fractures provided that the complete state of stress in the ground is known; that is, the value, orientation, and gradient of all three principal stresses. Of course, it is almost impossible to obtain this information, since it is extremely difficult to measure the state of stress at a point, let alone their space gradients. However, it is interesting to examine a few cases by assuming various extreme conditions as shown in Figure 7 for vertical fractures. In the first case ($K' = 0$) the least compressive stress (which must be oriented horizontally in order to produce the vertical fracture) is assumed to be constant with increasing depth. Under these conditions, the fracture will "see" a constant confining pressure over its entire area just as it does in a horizontal fracture. However, the fluid pressure inside the fracture is not uniform but is greater along its lower tip. The fracture will therefore propagate downward with less pumping pressure required as it gets longer, thereby causing the pump pressure to decrease rapidly during the course of the injection. In the second case ($K' = \gamma$), the vertical gradient of the least (horizontal) compressive stress is assumed to be exactly equal to the density factor of the fluid (0.433 psi/ft for water). Therefore, the two factors cancel each other and the pump pressure falls exactly as it would during the development of horizontal fracture. In the third case ($K' = \gamma_R$), the vertical gradient of the least compressive stress

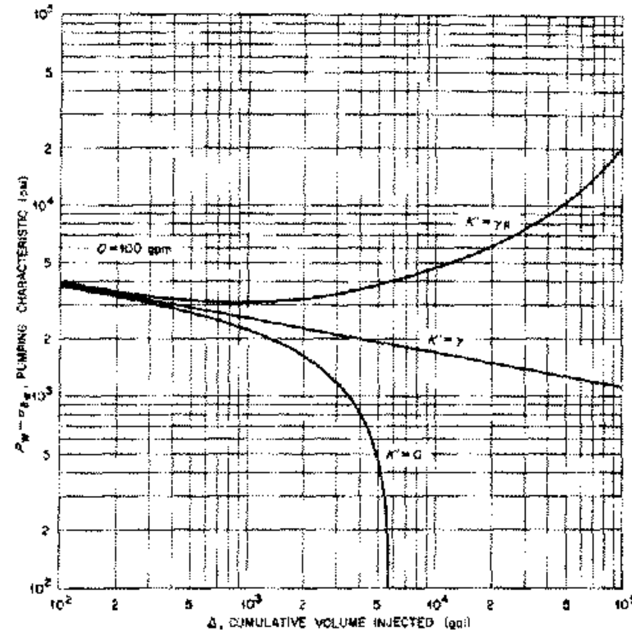

Figure 7. Pumping pressure as a function of volume injected for vertical fractures.

(which is horizontal) is assumed to be equal to the density factor of the rocks; that is, it is greater than that of the fracturing fluid. Therefore, the confining pressure at the lower edge of the fracture is greater than at the upper tip. The pressure gradient inside the fracture resulting from the static head of fluid will not be sufficient to make up the difference, and the fracture will be extended upward. However, as the fracture extends, more pressure will be required to lift the water so the pumping pressure will increase rapidly as the injection proceeds.

EFFECT OF INJECTION ON SURROUNDING ROCKS

Returning now to the discussion of the behavior of a circular horizontal fracture, it was seen that the average pressure in the fracture during the injection was greater than the minimum required to extend the fracture [(Eq. (1)] by an amount equal to the viscous loss resulting from the flow of the fluid in the fracture. This excess pressure is accommodated by an increase in the fracture width and by an increment of elastic deformation in rocks surrounding the fracture. When the pumping is stopped and the well "shut-in," the flow velocities, and consequently the hydraulic losses, will decrease very rapidly. The increment of stored elastic energy will then be recovered, causing the fracture to become thinner and larger until the static equilibrium pressure [Eq. (1)] is attained. During this period, the pressure at the well will be decreasing according to some complicated transient function. This interpretation may help explain some of the difficulties usually encountered in attempting to measure the "instantaneous shut-in pressure" in fracturing operations.

When the fracture has attained a stable condition and is no longer propagating, there will remain a slight internal pressure equal to the minimum fracture extension pressure which induces stresses in the surrounding rocks. In the case of waste-disposal operations, this pressure may be slightly larger than the true fracture extension pressure if the pressure transient is interrupted by the setting of the cement. The stresses induced in the surrounding rock can be calculated for the idealized situation and are shown as elastic strain contours in Figure 8. Examination of these induced stresses indicates that each injection produces a stress condition slightly more favorable for the initiation of a vertical rather than horizontal fracture in subsequent operations than existed prior to the injection. Furthermore, the location of the most adverse induced stresses is above the center of the grout sheets; that is, coincident with the injection well where the future operations will take place. The induced stresses along this line (Fig. 9) are very small for a single injection, being only a small fraction of the pressure in the fracture (P_o) which is itself less than 10 psi. However, in waste disposal operations, the effects of each injection would be cumulative; and, after a few score injections, appreciable stresses may be built up. A calculation (McClain, 1968a) was made of the capacity of the shale formation at Oak Ridge based on this analysis by assuming that all grout sheets would be located at the most adverse positions and that the most unfavorable original state of stress existed. This calculation indicated that the shales could accept at least 4×10^6 gal of waste or about 20 years at the present rate of disposal. Because of the ultra-conservative assumptions in that calculation, the

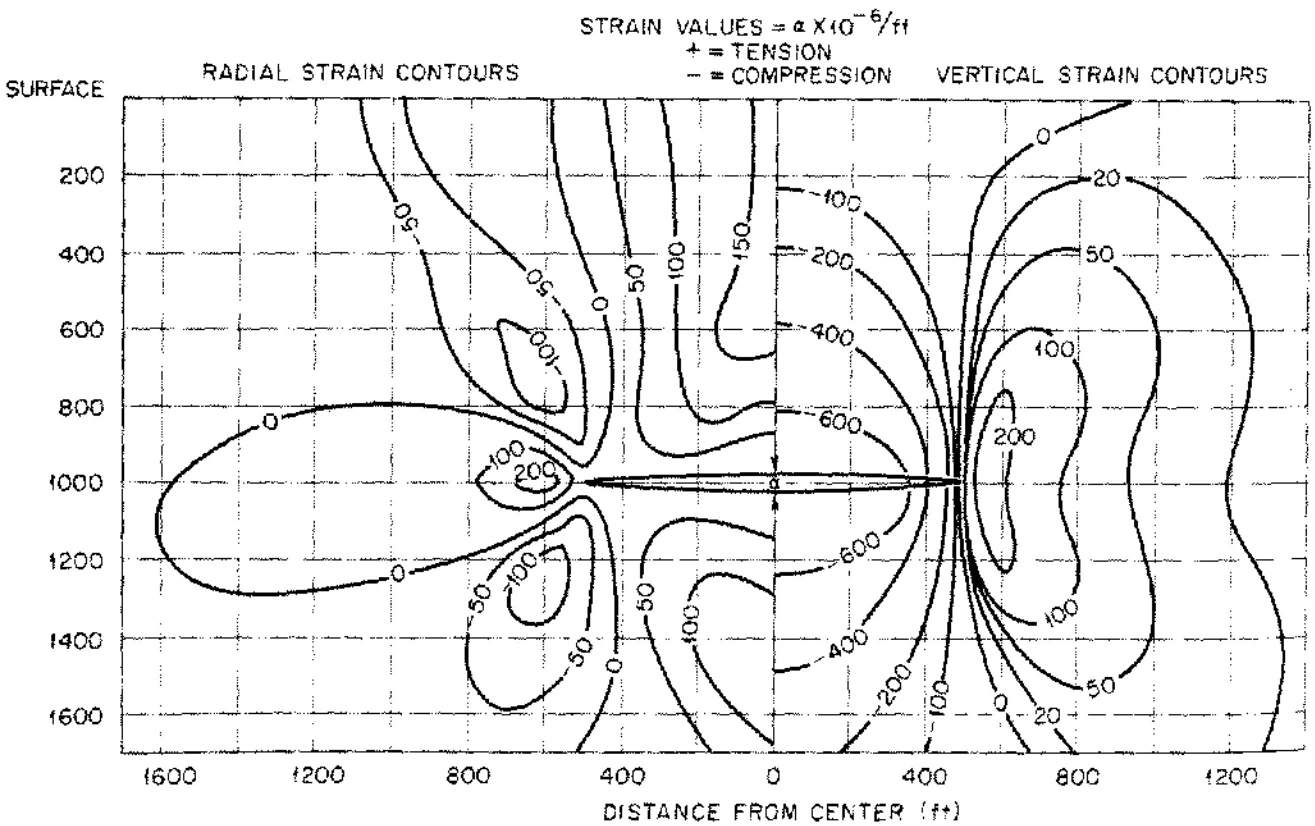

Figure 8. Radial and vertical elastic strain contours around a thin circular ellipsoidal fracture.

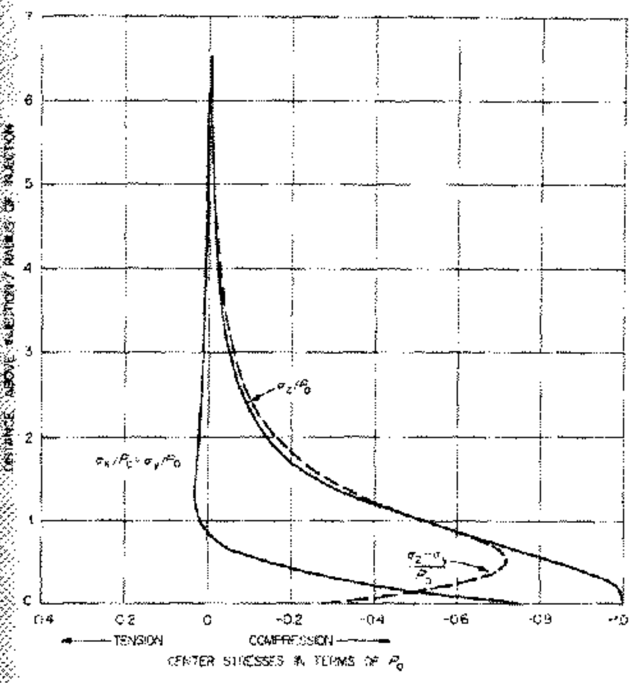

Figure 9. Induced stresses above center of circular ellipsoidal fracture in terms of internal pressure (P_0).

actual capacity at failure, if it is ever reached, is expected to be many times larger.

This treatment of the stresses induced by the fractures led to the consideration of the behavior of a fracture in a nonuniform stress field. At any place on the leading edge of a propagating fracture, the fluid pressure must be just equal to the confining stress (or overburden pressure for a horizontal fracture) plus the minimum fracture extension pressure. If the confining stress is less at some points around the crack periphery than at others, the fracture will be propagated only into those low confining stress areas. This feature is actually a corollary of the fundamental principle that the fracture will form in a direction perpendicular to the direction of the least compressive stress and can be stated more exactly as: Within the plane (or surface) normal to the least stress direction, the fracture will be propagated preferentially toward those regions where the stress perpendicular to the fracture is itself lowest. For example, if the minimum principal stress is everywhere vertical, a horizontal fracture will be developed. But, if the value of this vertical stress varies throughout the horizontal plane, being lower say to the north than to the south, the fracture will be extended to the north. Therefore, the shape of the resulting horizontal fracture can be used to provide an indication of the shape of vertical stress field. (See Figs. 2 and 3)

CONCLUSIONS

This paper has outlined the mechanics of hydraulic fracturing in shales as currently understood, based on analytical treatments and supported by experimental data. Because of the nature of the research and development program at Oak Ridge, it has frequently been possible to conduct tests or make measurements which otherwise would have been impossible. In spite of this advantage, the amount of good, reliable "hard" experimental data is still distressingly small. Consequently, this discussion represents only the current interpretation of the fracturing process and is subject to change.

One aspect of this analysis which especially needs further investigation is the fracture orientation control exercised by the bedding planes of the shale. This effect has been discounted in the analysis presented here because of the lack of a mechanism explaining how the bedding planes can exert a larger influence, even though both intuition and certain experimental results suggest that such a mechanism exists. For example, a recent paper (Shock and Davis, 1969) reported a fracturing operation in a thin potash seam where a horizontal fracture was developed under the control of a very poorly developed, almost nonexistent bedding plane.

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